DISCOVERY

NEVILLE de MESTRE

Laboratory mathematics

Computers were invented to help mathematicians perform long and complicated calculations more efficiently. By the time that a computing area became a familiar space in primary and secondary schools, the initial motivation for computer use had been submerged in the many other functions that modern computers now accomplish. Not only the mathematics department used computers, but every other discipline insisted on access for word processing, data storage and retrieval, and the Internet. In fact, many mathematics departments decided not to compete with other departments for use of the computer laboratory, and shifted to an emphasis on programmable calculators which were cheaper for every student to obtain and easier to carry to wherever they were needed by the class or at home.

For the 10 years that I have been preparing *Discovery* articles for *The Australian Mathematics Teacher*, I have noted the pioneering computer spirit of Hartley Hyde in his CACTUS articles which also appear in this journal. Hartley has endeavoured to show mathematics teachers that computers and spreadsheets could be used as an extremely helpful tool to aid students in their understanding of mathematics.

For 2005, my *Discovery* articles will be orientated with a strong emphasis on problem-solving using mathematics and hands-on manipulatives. I will endeavour to show that part of every mathematics curriculum in both primary and secondary schools should contain mathematics laboratory exercises. Computers, and spreadsheets in particular, are extremely useful hands-on manipulatives for all students, and so in this first *Discovery* article for 2005, I am going to tell you about exciting Discovery lessons in mathematics using a spreadsheet such as Microsoft Excel, which is available in every school.

I will assume that your students know how to enter formulae into cells, how to fill down or across, and how to use Chart Wizard in a basic sense to obtain diagrams of one or two functions. During the discovery lessons other features of the spreadsheet will become apparent, and pointed out at the appropriate moment.

As a warm-up exercise you could ask your students to find out how many cells are available for use in a single worksheet. Obtaining a count of the number of rows is easy, but the columns' count is a little harder. Rather than counting the columns by using the fact that there are 26 letters in the alphabet, your students could type 1 in cell A1, 2 in cell B1, and fill across to the end.

Next they could investigate arithmetic progressions by typing 2 in A1, 5 in A2, and filling down for about 10 cells. Chart Wizard can then be used to show the connection between arithmetic progressions and straight lines. An alternative approach should then be used with 2 typed in A1, then =A1+3 typed in A2, and again filling down. Finally any arithmetic progression can be displayed from the general use of absolute references. Cell C1 is named START and D1 is named DIFFERENCE. In cell C2 is typed 2, and D2 is typed 3, while A1 contains =$C$2 and A2 contains =A1+$D$2. After filling down column A, any arithmetic progression can be displayed immediately by simply changing the values in C2 and D2, even to negative numbers.

Geometric progressions can be treated in a similar fashion, but your students should now be encouraged to use the general spreadsheet approach. This time cell C1 is named START as before, while cell D1 is named RATIO. In cell C2 is typed 1, and in D2 is typed 2. Cell A1 still has =$C$2, but cell A2 now has =A1*$D$2.

After filling down for 10 cells and using Chart Wizard, the picture for geometric progressions emerges. If your students keep filling down for a large number of cells (ask them how many?), they will notice that the numbers change to scientific notation (1.34E+08) after a few rows. However, if the width of column A is increased or decreased, this can happen later or earlier than 1.34E+08. If filling down is continued, eventually the expression #NUM! is reached just after 9E+308, indicating that the computer's 'infinity' has been attained, that is, the computer will not specify values with more than 308 digits.

Your students will now be in a position to investigate population dynamics using a computer and, in particular, be able to observe chaotic behaviour and the ramifications of increasing the birth rate. Let x_N be the population at the beginning of the Nth year. Then x_0 is the starting population. If b denotes the cumulative birth and death rates, then for a linear model

$$x_{N+1} = bx_N$$

Starting with $x_0 = 100$ your students can use a worksheet to find out what happens to the population when $b < 1$, $b > 1$ and $b = 1$. They can also have a column for N so that they can ascertain when certain events occur (such as $x_N < 1$ when the population becomes extinct).

Next they could include a term denoting the limited availability of food and space resources, so that the birth rate diminishes as the population becomes too large. The simplest mathematical model for this is

$$x_{N+1} = \left[b\left(1000 - x_N\right)\right]x_n$$

where we have chosen the birth rate in [ ] to become zero when the population reaches 1000, but it could be at any suitable upper limit.

If this formula is scaled such that

$$X_N = \frac{x_N}{1000} \text{ and } B = 1000b$$

it becomes

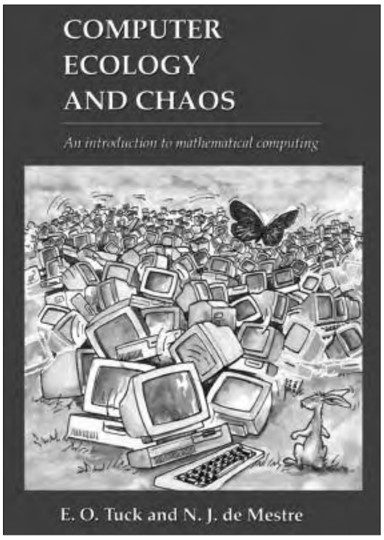
$$X_{N+1} = B\left(1 - X_N\right)X_N$$

which is a quadratic expression, and we note that for all populations with $0 < x_N < 1000$ we have $0 < X_N < 1$.

Your students can now enter N, X and B in cells A1, B1 and D1 respectively; then N=0 in cell A2, 0.5 in cell B2 and 0.1 in cell D2. Next enter 1 in cell A3, and =$D$2*(1-B2)*B2 in cell B3. After filling down columns A and B they can see what happens to the limited resources population model starting with $X_0 = 0.5$ (i.e., $x_n = 500$) and a birth rate of 0.1. It will not be much different from the linear population model. Try it also for $B = 0.9$ and different starting values bearing in mind that the starting value X_0 must be between 0 and 1 since $0 < X_N < 1$ for all N.

However, as B increases through 1 to 1.5, then 2, then 2.5, then 2.9, then 3, then 3.1, then 3.4, then 3.5, then 3.6, then 3.8, then 3.83, then 3.9, then 4, there are some strange behaviours occurring to the population. These behaviours should be discovered and discussed by your students. They include equilibrium or ultimate values, period doubling and chaos. A number of systematic Action Summaries for lessons associated with the behaviours can be obtained from the website www.bond.edu.au/it/staff/NevilleChaos.htm

Your students can generate diagrams for these behaviours using Chart Wizard. However, more sophisticated illustrations known as cobweb diagrams can also be generated using IF statements. How to do this with Excel, plus many other features of these laboratory exercises are contained in a CD developed by my colleague Professor Ernie Tuck and myself. This CD considers a spreadsheet version of our 1991 book (sadly, now out of print) *Computer Ecology and Chaos*, which was originally produced for users of BASIC. The website will tell you how to obtain a copy of the CD.



However, using the quadratic formula is only one example of population dynamics behaviour. Your students can consider many other non-linear formulae and investigate many other features, including culling for example, with the computer as a discovery tool.